The tensile failure of brittle matrix composites reinforced with unidirectional continuous fibres

R. W. DAVIDGE, A. BRIGGS
Materials Development Division, Harwell Laboratory, OX11 0RA, UK

The tensile failure strength of ceramic composites can be measured by tests in bending or in tension, but care must be exercised over the experimental conditions. The strength values obtained are dependent on the test method and specimen size. It is shown that differences between strengths measured in bend and tensile tests can be understood in terms of the statistical distribution of the strengths of individual fibres.

1. Introduction
The development of ceramic matrix composites reinforced with continuous fibres has reached the stage where applications in the engine, aerospace and military industries are under very active consideration. A broad and detailed understanding of the mechanical properties of these materials is thus essential to enable prediction of performance under service conditions. For well designed composites of this type the failure under tension is by a number of successive mechanisms. Deformation is elastic up to the ultimate strength of the matrix σ_{mu}, when the matrix cracks into a series of roughly parallel blocks normal to the tensile stress. The maximum strength of the composite σ_{cu} is controlled by fracture of the fibres. A further increase in strain then results in pullout of the fibres through the characteristic tough failure process.

Both σ_{mu} and σ_{cu} can be used for design purposes. The usual methods to evaluate these properties are through tests in bending or in tension. Bending tests are simple to perform but difficult to interpret: tensile tests are difficult to perform but simpler to interpret. The purpose of this paper is to consider bend and tensile data for these composites, the respective merits of the tests, and the relationship between the results obtained.

2. Test methods
In highly anisotropic materials it is crucial to ensure that the mode of failure of the specimen corresponds to that for which the test has been designed [1]. The shear strength, τ, on planes parallel to the fibres may by typically 50 MPa compared with σ_{cu} of 1000 MPa [2], and thus the suppression of shear failure demands a carefully designed test procedure.

In a three-point bend specimen, for example, of span l, breadth b, and thickness d, the ratio of tensile stress to shear stress σ_{cu}/τ is $2{:}l/d$. Typically therefore, in tests to measure σ_{cu}, an l/d ratio > 20 is used to suppress shear failure.

For tensile tests, a major problem is the attachment of the specimen to the test machine. This is commonly achieved by bonding end tabs to the specimen through which the load is applied by clamping. Shear stresses can readily lead to failure from the grips or specimen splitting. It is often necessary to contour the central portion of the specimen so that failure occurs well away from the gripped regions. The contour should be chosen so that the shear stresses in the contoured region are relatively small compared with the tensile stress [1].

3. Theory
The theory for aligned composites is relatively well-established and has been recently reviewed [3]. The idealized stress–strain curve in tension is shown in Fig. 1 represented by OACD. The matrix cracking stress is given by

$$\sigma_{mu} = \left(\frac{12\tau_i\gamma_m V_f^2 E_f(1-\nu^2)^2}{r\,V_m\,E_c E_m^2}\right)^{1/3} \frac{E_c}{1-\nu^2} \quad (1)$$

τ_i is the fibre–matrix frictional stress, γ_m the fracture surface energy of the matrix, V_f, V_m the volume fractions of fibre and matrix, E_f, E_m, E_c, the Young's moduli of the fibres, matrix and composite, ν Poisson's ratio and r the fibre radius. The material then extends at constant stress as the matrix cracks form. Ultimate failure occurs at a stress given by

$$\sigma_{cu} = \sigma_{fu} V_f \quad (2)$$

which is controlled by the breaking strength σ_{fu} of the fibres.

The ideal curve [4] (OACD) is observed in cases where there is a large difference between the stiffness of the fibres and the matrix, and where there is a low volume fraction of fibres. In the materials considered here the differences in Young's modulus between fibres and matrix are relatively small and the volume fraction of fibres is relatively high. The observed curve (solid line) thus usually shows a change of slope at the cracking stress and a gradual reduction in stiffness near to σ_{cu}, where the fibres begin to break.

The same general type of behaviour is expected in bending. However, at stresses greater than σ_{mu}, part of the tensile side of the beam becomes cracked and

0022–2461/89 $03.00 + .12

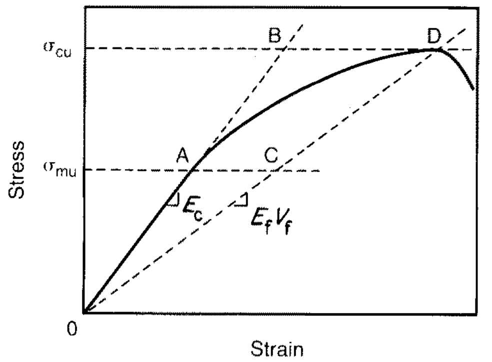

Figure 1 Idealized stress–strain behaviour of a brittle matrix composite in tension.

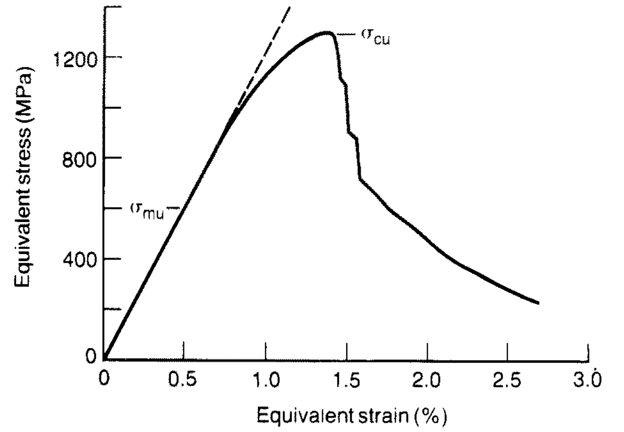

Figure 2 Stress–strain curve in bending for SiC fibre in glass composite. Work of fracture = $70\,\mathrm{kJ\,m^{-2}}$.

the beam must be considered as a composite beam with two layers of different stiffness.

4. Experimental data

For illustrative purposes data are summarized [2, 5] for a borosilicate glass reinforced with Nicalon silicon carbide continuous fibres, tested both in bending and in tension, Table I. Stress–strain curves measured in bending and tension are shown in Figs 2 and 3. The quoted bending stresses are those derived from simple beam theory, ignoring the matrix cracking effect which leads to a less stiff layer on the tensile side of the specimen. The stresses in tension relate to the cross-section area of the original composite, even after matrix cracking, where all the load is borne by the fibres.

The scatter in strengths for bend tests of 100 samples is shown in Fig. 4 where the Weibull modulus is 30. Relatively fewer (~ 10) tests have been conducted in tension and the Weibull modulus is ~ 12.

Some details of the failure of a bend specimen are shown in Fig. 5 [2]. Microscopic observations of the tensile specimens indicate that they often failed in a mixture of shear and tension, with an interaction with the gripped region; this contributes to the low value of the Weibull modulus found in tension. Fig. 6 shows a tensile failure of a specimen failing near its centre [5].

5. Discussion

We can now discuss the absolute results obtained in the above tests and also the differences between bending and tensile data. It is convenient first to consider geometrical effects so that the true values of the bend and tensile strengths can be obtained. Then the effects of specimen size and stress distribution can be computed.

5.1. Effects of matrix cracking on the true bend strength

When the stress in a bend specimen is greater than σ_{mu} the matrix will crack. The cracked matrix region will extend as deformation proceeds. The strain varies linearly across the beam and when the beam is all of the same stiffness the stress varies in like manner. When the tensile surface becomes cracked the stiffness is reduced and thus the true stress on the surface is less than expected from the naive calculation. For a very thin cracked surface layer the tensile stress is reduced by a factor $E_f V_f / E_c$. As the cracked region grows the overestimate becomes less. Aveston *et al.* [6] have computed the overestimate factor precisely for the case of a beam in pure bending where the load–deflection curve corresponds to the ideal case in Fig. 1. Their results show that the factor at the ultimate failure point for the current material is 1.1. The facts that the load–deflection curve is slightly different from the ideal case in Fig. 1, and that three point rather than pure bending is used, do not significantly affect this value. We conclude, therefore, that the true value of the bend strength is $1250/1.1 = 1140\,\mathrm{MPa}$ (equivalent to $700 \times 0.49/0.45 = 760\,\mathrm{MPa}$ at the same V_f).

TABLE I Summary of test data for borosilicate glass reinforced with silicon carbide fibres [2, 5]

	l (mm)	b (mm)	d (mm)	V_f	σ_{mu} (MPa)	σ_{cu} (MPa)	m for σ_{cu}
Bend	40	6	0.8	0.49	600	1250	30
Tension	20	6	1.5	0.45	500	600	12

5.2. The true tensile stress

A proportion of the samples tested in tension failed by a mixture of tensile fracture plus shear along planes parallel to the fibres into the grip regions. We believe, therefore, that the true tensile strength of the material is towards the upper end of the observed range of tensile results, say 700 MPa.

5.3. Stress distribution and specimen size effects

In considering effects of stress distribution and specimen size on the strength of ceramic specimens it is

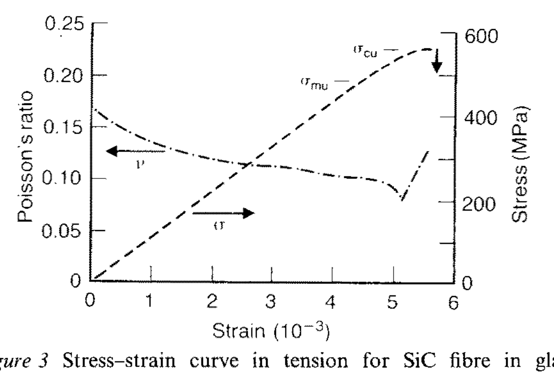

Figure 3 Stress–strain curve in tension for SiC fibre in glass composite.

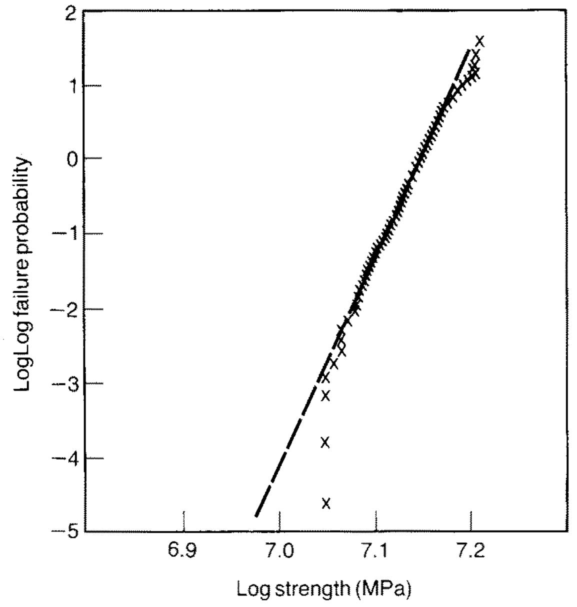

Figure 4 Statistical variation in ultimate bend strength. Weibull modulus = 30, mean strength = 1.25 GPa.

common to use arguments based on a Weibull distribution of strengths with an assumed flaw population throughout the volume of the material. The usual relationship relating three-point bend strength to tensile strength for specimens of equal volume is [3]

$$\frac{\sigma_b}{\sigma_t} = [2(m + 1)^2]^{1/m} \tag{3}$$

A simple mechanistic application of Equation 3 using $m = 30$ gives $\sigma_b/\sigma_t = 1.29$ compared with the observed value of 1140/760 = 1.50. However, apart from this discrepancy, we believe that this simple approach cannot be applied to the current material because ultimate failure is associated with the fracture of a large number of fibres rather than from a single flaw. Furthermore the material is not notch sensitive, so that "weakest link" Weibull theory seems inappropriate.

As an alternative explanation we propose that calculation of the ultimate failure stress of the composite requires knowledge of the distribution of facture strengths of the individual fibres, the stress variation along the fibre, and the way in which stress is transferred from a broken fibre to its neighbours.

The classical problem of the relationship between the strength of a bundle of fibres and the strength of individual fibres has received a great deal of sophisticated mathematical treatment in the literature. The simplest case is for a series of parallel fibres perfectly clamped at their ends such that each fibre is subjected to an equal strain. It is assumed that the strength distribution of individual fibres is described by a Weibull function. As the fibres break successively under an increasing total load, the load on each broken fibre is shared equally between the surviving unbroken fibres. This process continues until the surviving fibres can no longer collectively sustain the applied load whereupon they all fracture. The original analysis was due to Daniels [7] and this was subsequently related by Coleman [8] to a Weibull distribution of strength. More recent studies have extended this analysis from the simple (equal load sharing) situation to a local load sharing case where the load sustained by a broken fibre is preferentially borne by its immediate

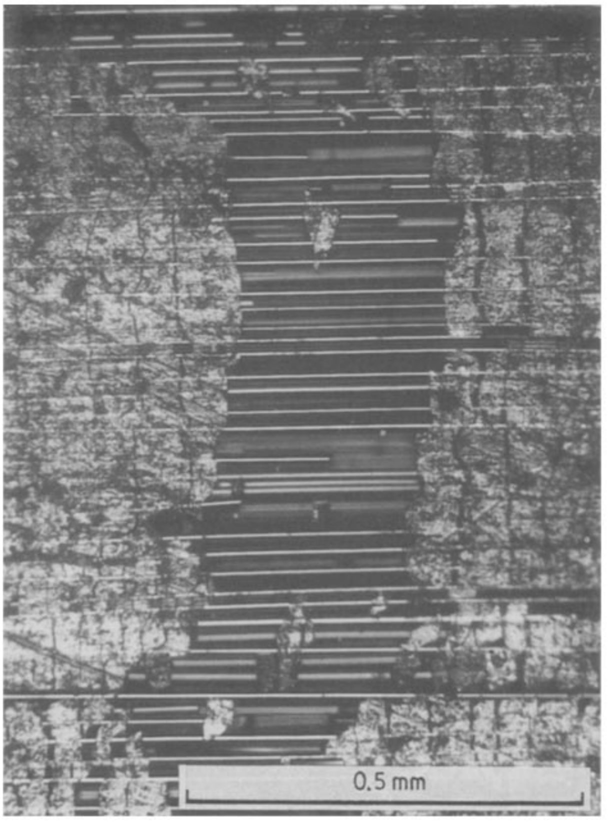

Figure 5 Specimen tested in bending to and beyond failure (nominal strain 2.5%).

Figure 6 Specimen tested in tension to just beyond σ_{cu}.

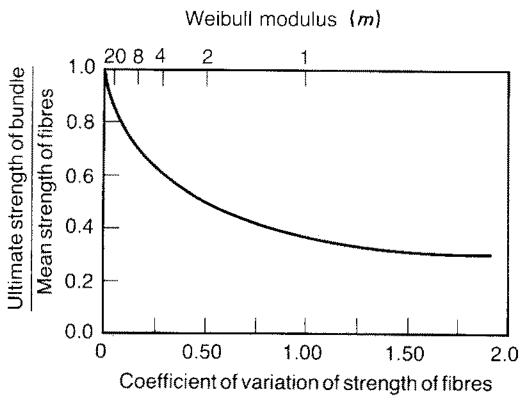

Figure 7 The ratio of ultimate fibre bundle strength to mean fibre strength as a function of variation of the fibre strength.

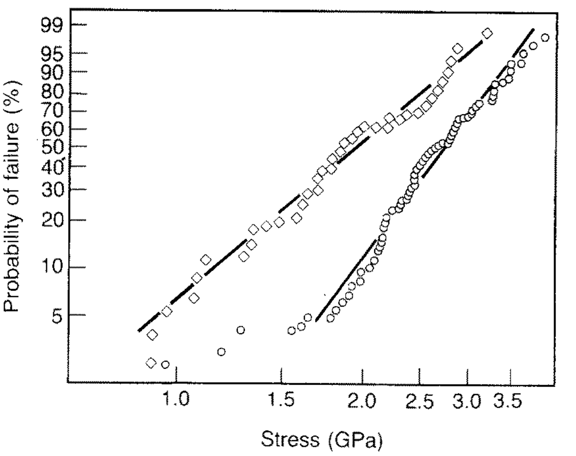

Figure 8 Statistical variation in strength of Nicalon SiC fibres of 10 (○, $m = 5.4$) and 100 (□, $m = 3.6$) mm length.

neighbours; and for other cases of practical interest where the strength of the fibres is time dependent, and where there are deviations from ideal parallel fibre geometry.

We restrict discussion here to the classical case in the belief that the current system approximates reasonably well to this, and in the absence of a more detailed understanding of the total fracture process. The results of Coleman, Fig. 7, show how the ratio of the strength of the bundle to the mean strength of individual filaments varies with the Weibull modulus or the coefficient of variation in strength of the individual fibres. The strength of the fibre bundle is always less than the average strength of the individual fibres, and in some cases appreciably so. Note that the mean strength of the fibre bundle does not depend on the number of fibres in the bundle. On the other hand the mean bundle strength does depend on the length of the fibres which controls the strength of the individual fibres.

The scatter in the strength for individual fibres is controlled by the variation in flaw size, each fibre breaking at the stress to propagate the largest flaw. In contrast, the stress to break the fibre bundle is a cooperative effect involving the simultaneous fracture of many fibres. Thus the scatter in strength of fibre bundles is much less than that for the individual fibres and decreases with increasing number of fibres in the bundle. This is an agreement with the small strength variation observed for the ultimate composite strength in bending, Fig. 4, compared with that for the fibres, Fig. 8.

It is difficult to determine the strength of the fibres in the composite after they have been subjected to the complete fabrication cycle. Both physical and chemical degradation may occur, and almost certainly will unless the process has been carefully optimized. The strength of single fibres of Nicalon SiC has been measured [9] for 10 and 100 mm lengths, Fig. 8. The mean strength is 2.6 GPa for 10 mm fibres and 1.9 GPa for 100 mm fibres. The Weibull modulus is 4 to 5. For an ideal distribution of flaws throughout the volume of the fibres one would expect, from the ratio of strengths ($\sigma_1/\sigma_2 = 1.37$) for the two lengths ($l_2/l_1 = 10$), a Weibull modulus $m = 7$ using

$$\sigma_1/\sigma_2 = (l_2/l_1)^{1/m} \tag{4}$$

This is higher than the values in Fig. 8 but for 2 mm long fibres the Weibull modulus is reported [9] to be higher at 8. There is thus some danger in too rigorous an application of statistical strength data for fibres, but some general considerations are useful.

The mean strength of 20 mm long fibres (the length of the tensile specimens) is estimated from Fig. 8 as 2.4 GPa. The bundle strength–mean strength ratio from Fig. 7 is 0.65. V_f is 0.45. Thus, from Equation 2 $\sigma_{cu} = 2400 \times 0.65 \times 0.45 = 700$ MPa, which corresponds to the experimental value.

To relate the bend and tensile strengths requires modification of Equation 3 because the stressed volume integral for bending relevant to the current material depends only on the stress distribution along the length of the specimen and not throughout the thickness (because the fibre bundle strength does not depend on the number of fibres). Equation 3 reduces to

$$\frac{\sigma_b}{\sigma_t} = (m + 1)^{1/m} \tag{5}$$

where m relates to the strength of single fibres not the strength of the bulk material. The experimentally determined bend and tensile strengths (Table I) can now be compared noting that the fibre volume fraction and the specimen sizes are slightly different, according to

$$\frac{\sigma_b}{\sigma_t} = \frac{V_{f(b)}}{V_{f(t)}}\left(\frac{l_{(t)}}{l_{(b)}}(m + 1)\right)^{1/m} \tag{6}$$

The ratio from Equation 6 using $m = 4$ is 1.43 compared with the experimental ratio of 1.50. The values are sufficiently close to warrant further experimental work.

6. Conclusions

The ultimate strengths of composites measured in bending and tension are significantly different. The bend strength is slightly overestimated because matrix cracking causes a shift of the neutral axis of the beam. The tensile strength may be underestimated because of mixed tensile–shear failure.

The application of simple equal load sharing theory to the strength of fibre bundles is consistent with the experimental observations for the composites: (i) the very low scatter in the bend strength values and (ii) the absolute and relative values of the bend and tensile strengths.

Further work is clearly necessary to verify whether this simple approach is more generally applicable. Important predictions are that for tests conducted with different specimen dimensions: (i) strength should be very dependent on the dimension parallel to the fibres and (ii) strength should not vary with cross-section normal to the fibres.

Acknowledgements
Financial support from the Department of Trade and Industry through the Harwell Materials Engineering Centre is gratefully acknowledged. Thanks are due to Dr D. C. Phillips for useful discussions and comment on the manuscript.

References
1. D. C. PHILLIPS and R. W. DAVIDGE, *Br. Ceram. Trans. J.* **85** (1986) 123.
2. D. M. DAWSON, R. F. PRESTON and A. PURSER, *Ceram. Eng. Sci. Proc.* **8** (1987) 815-821.
3. R. W. DAVIDGE, "Fracture of Non-metallic Materials", edited by K. P. Herman and L. H. Larsson (ECSC, EEC, EAEC, Brussels and Luxembourg, 1987) pp 95-115.
4. J. AVESTON, G. A. COOPER and A. KELLY, Proceedings Conference on The Properties of Fibre Composites, (IPC Sci and Tech. Press, Guildford UK, 1971).
5. D. M. DAWSON, A. BINSTEAD and F. K. KO, *Ceram. Eng. Sci. Proc.* in press.
6. J. AVESTON, R. A. MERCER and J. M. SILWOOD, Composites – Standards, Testing and Design. NPL Conf. Proc. 8-9 April (1974), 93-103. (IPC Sci. & Tech. Press, London, 1974).
7. H. E. DANIELS, *Proc. R. Soc.* **A183** (1945) 405.
8. B. D. COLEMAN, *J. Mech. Phys. Solids*, **7** (1958) 60-70.
9. C-H. ANDERSON and R. WARREN, *Composites*, **15** (1984) 16–24.

Received 30 June
and accepted 1 November 1988